THE NEGLECTED LOGARITHM GRAPH

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Two problems

Problem 1.

Finding when Compound Annual Rate of Growth (CAGR) changes.

- Harder than it looks because CAGRs fluctuate from year-to-year

Problem 2.

Plotting data that differ by a factor of 100 or more on a common grid.

Compound Annual Growth Rate

$$CAGR = (Y_{t+n}/Y_t)^{(1/n)} - 1$$

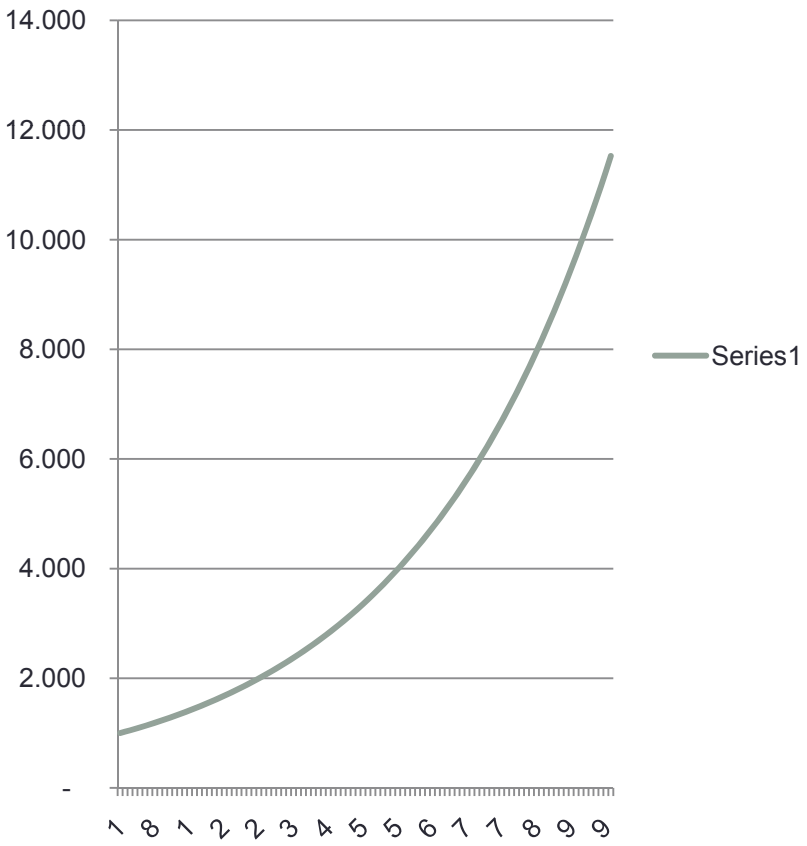
Issues

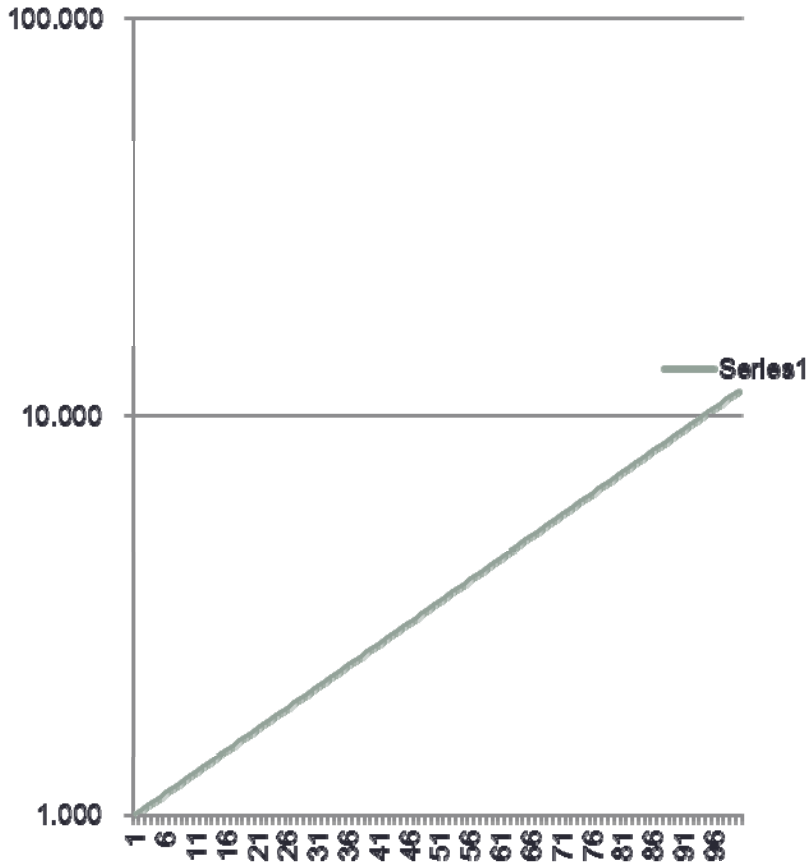
- Disruptions occasionally occur
- Using only two data points may produce a misleading result.
  - too high to represent some sub-intervals &
  - too low to represent others
- It is not necessarily a “best” average value (i.e., it does not minimize the sum of squared deviations from mean)

Solution

- Break the framing period (length of data string) into intervals where CAGR is constant.
  - But, how to choose the break points?
- Convert data into logarithms (‘*logs’*) & put on a chart
  - Periods of constant CAGR appear as straight lines.
- The chart on the next slide shows this effect with a hypothetical variable that is constructed to grow at 2.5% per year (left chart) & a plot of its log (right chart)

Plot of 2.5% growth: Actual & log values

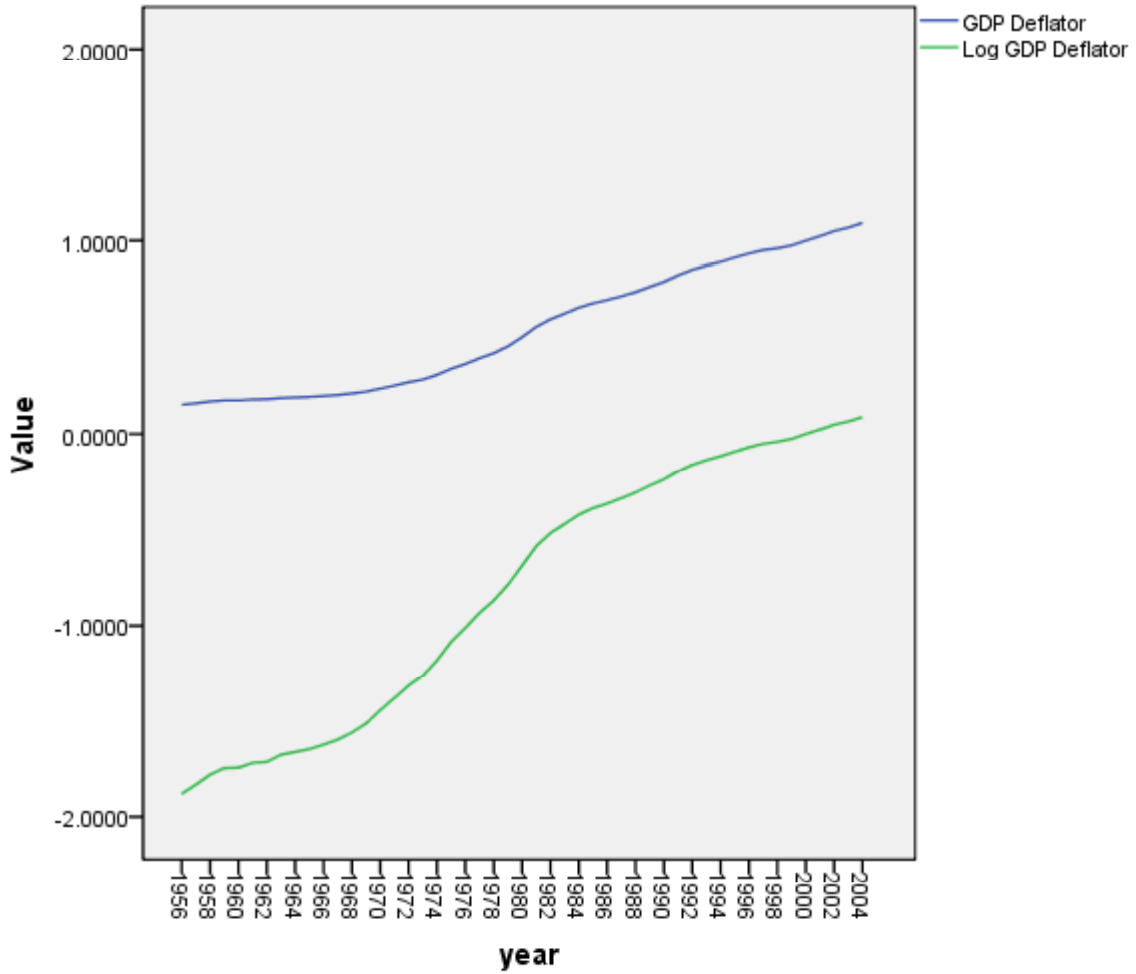




Real example

- The next slide shows the GDP deflator (top line) & its natural logarithm (bottom line)
- The top line shows only one break (1983)
- The bottom line clearly shows three (1960, 1968, 1983)

GDP deflator & its log



Break points

- Start in 1953 - Elimination of price controls & pent-up excess demand kicks off inflation
- 1960 - Excess demand eliminated naturally
- 1968 – New spending on the Great Society & Vietnam War finally causes inflation to pick up
  - Spending turns sharply up two years earlier but inflation lags
- 1983 – Price increases return to normal after the Fed slows down demand with historically high interest rates
  - The prime rate hit a peak of 20% two years earlier.

4 periods of (almost) constant CAGR

interval	n	CAGR
1953-1960	7	3.6%
1960-1968	8	2.3%
1968-1983	15	7.5%
1983-2005	22	2.7%
1953-2005	52	4.1%

Another advantage of logs

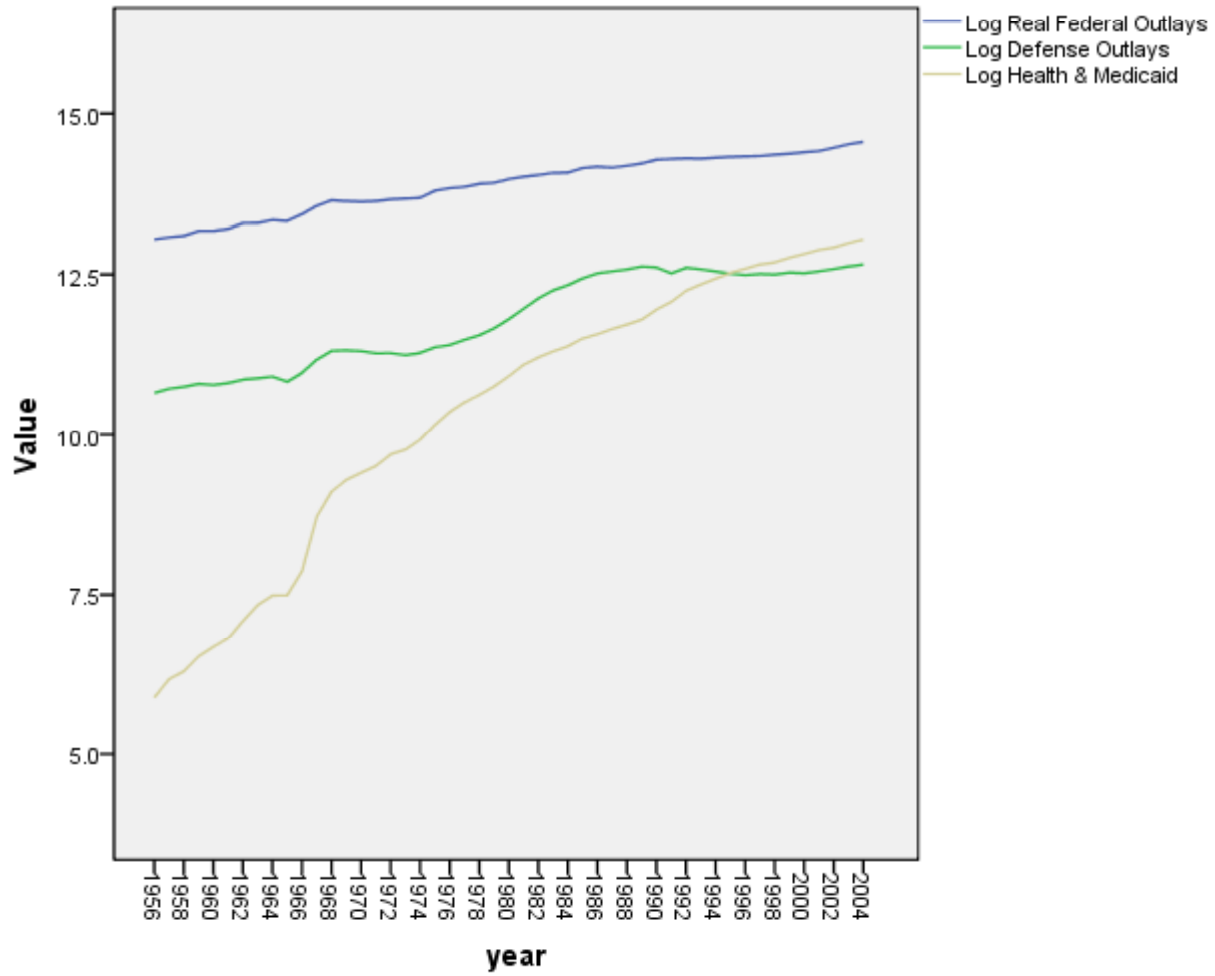
- Presenting data of vastly different size on the same grid with a common scale on the vertical axis

Difference between adjacent rows is a factor of 12.

Scale conversion for next chart

Natural Log	Bn of Current $
15.0	3,250.0
12.5	270.0
10.0	23.0
7.5	1.8

Logs of real total federal outlays with defense & health/Medicare components



Caveat

- Logs are defined for positive numbers only.
- Logs cannot represent zero or negative numbers.

THE END